Topic: Values that make the function differentiable

Question: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} 3x^2 & \text{if } x > 1 \\ bx^2 - a & \text{if } x \leq 1 \end{cases}$$

Answer choices:

A $a = 3$ and $b = 0$

B $a = 0$ and $b = 0$

C $a = 0$ and $b = 3$

D $a = 3$ and $b = 3$

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Solution: C

In order for the function to be differentiable, it must be both continuous, and differentiable. That means that both pieces of the function, when they meet at the "break point" of $x = 1$, must have the same value, and also must have the same slope (value of the derivative).

To make them have the same value at the "break point", and thereby make the function continuous, first find $f(1)$ for both parts of the function.

$$f(1) = 3(1)^2$$

$$f(1) = 3$$

and

$$f(1) = b(1)^2 - a$$

$$f(1) = b - a$$

Because the function has to be continuous, these two values have to meet each other, so

$$3 = b - a$$

This also means that

$$\lim_{x \to 1^+} f(x) = 3 \text{ and } \lim_{x \to 1^-} f(x) = b - a$$

To make sure that the slope of each side of the function is the same at the break point of $x = 1$, we'll take the derivative of both limits, and evaluate them at $x = 1$.

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$$\lim_{x\to 1^+} f'(x) = 6x$$

$$\lim_{x\to 1^+} f'(1) = 6(1)$$

$$\lim_{x\to 1^+} f'(1) = 6$$

and

$$\lim_{x\to 1^-} f'(x) = 2bx$$

$$\lim_{x\to 1^-} f'(1) = 2b(1)$$

$$\lim_{x\to 1^-} f'(1) = 2b$$

Setting these values equal to one another to force the slope of each piece to be the same gives

$$6 = 2b$$

$$b = 3$$

Use the other equation $3 = b - a$ and $b = 3$ to find a.

$$3 = b - a$$

$$3 = 3 - a$$

$$a = 0$$

The values of a and b that make the function differentiable are $a = 0$ and $b = 3$.

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Topic: Values that make the function differentiable

Question: Which values of a and b would make the function differentiable?

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x > 3 \\ 4x^2 - 2ax + b & \text{if } x \leq 3 \end{cases}$$

Answer choices:

A $a = 22$ and $b = 9$

B $a = 22$ and $b = 22$

C $a = 9$ and $b = 9$

D $a = 9$ and $b = 22$

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Solution: D

In order for the function to be differentiable, it must be both continuous, and differentiable. That means that both pieces of the function, when they meet at the "break point" of $x = 3$, must have the same value, and also must have the same slope (value of the derivative).

To make them have the same value at the "break point", and thereby make the function continuous, first find $f(3)$ for both parts of the function.

$$f(3) = (3)^2 - 5$$

$$f(3) = 4$$

and

$$f(3) = 4(3)^2 - 2a(3) + b$$

$$f(3) = 36 - 6a + b$$

Because the function has to be continuous, these two values have to meet each other, so

$$4 = 36 - 6a + b$$

This also means that

$$\lim_{x \to 3^+} f(x) = 4 \text{ and } \lim_{x \to 3^-} f(x) = 36 - 6a + b$$

To make sure that the slope of each side of the function is the same at the break point of $x = 3$, we'll take the derivative of both limits, and evaluate them at $x = 3$.

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$$\lim_{x\to 3^+} f'(x) = 2x$$

$$\lim_{x\to 3^+} f'(3) = 2(3)$$

$$\lim_{x\to 3^+} f'(3) = 6$$

and

$$\lim_{x\to 3^-} f'(x) = 8x - 2a$$

$$\lim_{x\to 3^-} f'(3) = 8(3) - 2a$$

$$\lim_{x\to 3^-} f'(3) = 24 - 2a$$

Setting these values equal to one another to force the slope of each piece to be the same gives

$$6 = 24 - 2a$$

$$a = 9$$

Use the other equation $4 = 36 - 6a + b$ and $a = 9$ to find b.

$$4 = 36 - 6(9) + b$$

$$4 = -18 + b$$

$$b = 22$$

The values of a and b that make the function differentiable are $a = 9$ and $b = 22$.

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